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NATURAL CONVECTION IN AN INCLINED SQUARE CHANNEL

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Abstract—Experimental and numerically computed values for the Nusselt number for natural convection heat transfer in an inclined, square channel maintained at uniform temperature on one inclined side and at a lower uniform temperature on the opposing side were determined and found to be in agreement. The maximum rate of heat transfer was found both theoretically and experimentally to occur at about 50 degrees of inclination. The minimum heat transfer rate and a change in the orientation of the two-dimensional roll-cells were found experimentally to occur at an inclination of about 10 degrees.

NOMENCLATURE

c_p, specific heat;
g, acceleration due to gravity;
h, local heat transfer coefficient $= q/(\theta_h - \theta_c)$;
H, height and width of square channel;
k, thermal conductivity;
Nu, Nusselt number $= qH/k(\theta_h - \theta_c)$;
$\overline{Nu}$, average Nusselt number $= \dfrac{1}{H}\displaystyle\int_0^H Nu\,dx$;
p, pressure;
p^*, pressure perturbation above the static state;
p_0, static pressure;
Pr, Prandtl number $= c_p\mu/k$;
q, heat flux density;
Ra, Rayleigh number $= \rho_0^2 g c_p \beta(\theta_h - \theta_c)H^3/k\mu$;
t, time;
T, dimensionless temperature $= (\theta - \theta_0)/(\theta_h - \theta_c)$;
u, velocity component in the x-direction $= \partial\Phi/\partial y$;
U, dimensionless velocity in the X-direction $= uH/\kappa$;
v, velocity component in the y-direction $= -\partial\Phi/\partial x$;
V, dimensionless velocity in the Y-direction $= vH/\kappa$;
x, distance from left side of channel;
X, dimensionless coordinate $= x/H$;
ΔX, finite grid-size in X-direction used in numerical analysis;
y, distance from top of the channel;
Y, dimensionless coordinate $= y/H$;
ΔY, finite grid-size in Y-direction used in numerical analysis.

Greek letters

β, volumetric coefficient of expansion with temperature;
ζ, vorticity $= \partial v/\partial x - \partial u/\partial y$;
$\hat{\zeta}$, dimensionless vorticity $= \zeta H^2/\kappa$;
θ, temperature;
κ, thermal diffusivity $= k/\rho c_p$;
μ, viscosity;
ρ, density;
τ, dimensionless time $= t\kappa/H^2$;
Φ, stream function;
Ψ, degree of inclination of the hot plate from the horizontal plane;
Ω, electric resistance [ohm].

Mathematical symbols

$\dfrac{D}{Dt}$, substantial derivative $= \dfrac{\partial}{\partial t} + u\dfrac{\partial}{\partial x} + v\dfrac{\partial}{\partial y}$;
∇^2, two-dimensional Laplacian $= \dfrac{\partial^2}{\partial X^2} + \dfrac{\partial^2}{\partial Y^2}$.

Subscripts

0, value at mean temperature;
h, value at hot plate;
c, value at cold plate.

1. INTRODUCTION

FREE convection from an inclined plate has been studied extensively (see for example Kierkus [1]). In spite of its importance the effect of inclination on natural convection in a confined channel does not appear to have received comparable attention. Holst and Aziz [2] reported theoretical results for natural convection in porous media in both horizontal and inclined regions but did not compare their results with experiments for the inclined case. Aziz *et al.* [3] studied experimentally the effect of inclination on natural convection in porous media in a confined box but they inclined the long dimension of the box and their result is not directly related to the case studied herein. Ozoe and Churchill [4, 5] recently reported on the effect of several boundary conditions on the orientation of roll-cells in a rectangular channel heated from below. They found experimentally that the preferred mode of natural convection in a long, confined, square channel heated on one vertical side and cooled on the other with the horizontal surfaces insulated was a single, long, two-dimensional roll-cell with its axis in the long dimension of the channel. On the other hand the preferred mode in a channel heated from below and cooled from above was found to consist of a series of two-dimensional roll-cells with their axes horizontal and perpendicular to the long dimension of the channel. Complete numerical solutions were developed for this latter mode.

In the current investigation the heat transfer rate and mode of natural convection were determined both experimentally and theoretically for the system shown in Fig. 1. The upper surface of the square channel

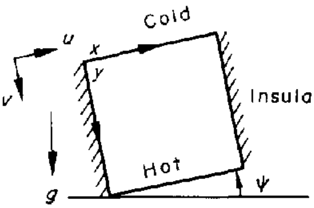

FIG. 1. Orientation of channel.

was maintained at a uniform low temperature, the lower surface at a uniform higher temperature and the other two sides were insulated. The channel was then rotated from the horizontal plane while keeping the long dimension horizontal.

2. MATHEMATICAL MODEL

Under the usual Boussinesq assumptions the fluid in a long, square, inclined channel can be described by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\rho\frac{Du}{Dt}=-\frac{\partial p}{\partial x}-\frac{\partial}{\partial x}\left(-2\mu\frac{\partial u}{\partial x}\right)-\frac{\partial}{\partial y}\left(-\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)-\rho g\sin\Psi \tag{2}$$

$$\rho\frac{Dv}{Dt}=-\frac{\partial p}{\partial y}-\frac{\partial}{\partial y}\left(-2\mu\frac{\partial v}{\partial y}\right)-\frac{\partial}{\partial x}\left(-\mu\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right)+\rho g\cos\Psi \tag{3}$$

$$\rho c_p\frac{D\theta}{Dt}=k\left(\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right) \tag{4}$$

$$\rho=\frac{\rho_0}{1+\beta(\theta-\theta_0)} \tag{5}$$

$$\theta_0=(\theta_h+\theta_c)/2 \tag{6}$$

and boundary conditions:

$$\theta(x,0,t)=\theta_c$$

$$\theta(x,H,t)=\theta_h$$

$$\frac{\partial\theta}{\partial x}(0,y,t)=\frac{\partial\theta}{\partial x}(H,y,t)=0$$

$$u(0,y,t)=u(H,y,t)=u(x,0,t)=u(x,H,t)=0$$

$$v(0,y,t)=v(H,y,t)=v(x,0,t)=v(x,H,t)=0.$$

If there is no motion in the fluid, the total pressure is equal to the static pressure. In general the total pressure may be represented as the sum of the static pressure and a perturbation:

$$p=p_0+p^*. \tag{7}$$

At $u=v=0$

$$-\frac{\partial p_0}{\partial x}-\rho_0 g\sin\Psi=0 \tag{8}$$

and

$$-\frac{\partial p_0}{\partial y}+\rho_0 g\cos\Psi=0. \tag{9}$$

Then

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}-g\sin\Psi=g\beta(\theta-\theta_0)\sin\Psi-\frac{1}{\rho_0}\frac{\partial p^*}{\partial x} \tag{10}$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y}+g\cos\Psi=-g\beta(\theta-\theta_0)\cos\Psi-\frac{1}{\rho_0}\frac{\partial p^*}{\partial y}. \tag{11}$$

Insertion of equations (8)–(11) into the momentum equation, introduction of the vorticity and stream functions defined as:

$$\zeta=-\nabla^2\Phi=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y} \tag{12}$$

and elimination of the pressure terms by taking derivatives of the momentum equations and subtracting, reduces equations (1)–(3) to

$$\frac{D\zeta}{Dt}=-g\beta\left(\frac{\partial\theta}{\partial y}\sin\Psi+\frac{\partial\theta}{\partial x}\cos\Psi\right)+\frac{\mu}{\rho_0}\left(\frac{\partial^2\zeta}{\partial x^2}+\frac{\partial^2\zeta}{\partial y^2}\right). \tag{13}$$

Dedimensionalizing equations (13) and (4) and the boundary conditions by the procedure of Hellums and Churchill [6]:

$$\frac{1}{Pr}\frac{D\zeta}{D\tau} = -Ra\left(\frac{\partial T}{\partial Y}\sin\Psi + \frac{\partial T}{\partial X}\cos\Psi\right) + \nabla^2\zeta \qquad (14)$$

$$\frac{DT}{D\tau} = \nabla^2 T \qquad (15)$$

with

$$T(X,0,\tau) = -1/2$$

$$T(X,1,\tau) = 1/2$$

$$\frac{\partial T(0,Y,\tau)}{\partial X} = \frac{\partial T(1,Y,\tau)}{\partial X} = 0$$

$$U(0,Y,\tau) = U(1,Y,\tau) = U(X,0,\tau) = U(X,1,\tau) = 0$$

$$V(0,Y,\tau) = V(1,Y,\tau) = V(X,0,\tau) = V(X,1,\tau) = 0.$$

3. NUMERICAL RESULTS FOR THE MATHEMATICAL MODEL

The mathematical model was solved numerically by the same procedure as used by Ozoe and Churchill [7]. The steady-state solutions were obtained as the limiting values for transient calculations from the steady, conductive state or following a step-change of Rayleigh number from a convective, steady state. Numerical instability occurred right at the beginning of the step-response with some initial conditions but was surmounted in all cases by the choice of some other steady-state as the initial condition.

The Rayleigh number was varied over a range for a series of inclinations. The range of computation was limited to $Ra \leqslant 8000$ yielding $Nu \leqslant 3$. The results are applicable only for laminar natural convection in this range. $Pr = 10$ was chosen for the calculations. The results would be expected to be valid for all $Pr \gg 1$. The computer used was the FACOM 230-60 (Fujitu Comp.) at the Kyoto University Computer Center. The Average computing time was 0·67 cpu s per time-step of integration. Usually 200–250 steps were required for convergence to the steady state. Ozoe and Churchill [7] studied the effect of finite grid-size on the numerical error for a similar model. They found that excessive computation was required for convergence but that the results at several finite grid-sizes could be extrapolated to zero grid-size with reasonable confidence. The Nusselt numbers obtained herein were corrected in this same way.

The corrected Nusselt numbers calculated from the heat flux at the middle height of the channel are plotted vs inclination in Fig. 2. The maximum value of the Nusselt number appears to occur at about 50 degree of inclination for all Rayleigh numbers. The Nusselt number can therefore be cross-plotted vs Rayleigh number with degree of inclination as only a minor parameter as indicated in Fig. 3.

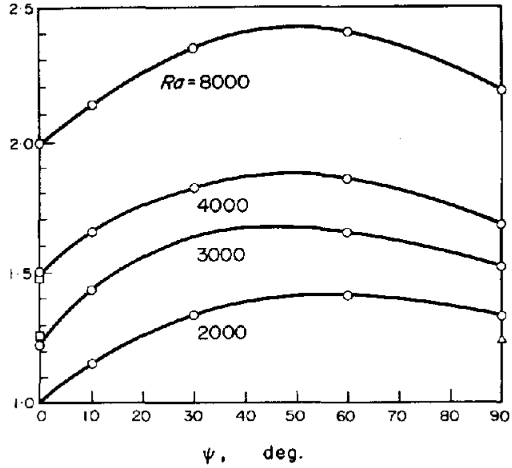

FIG. 2. Computed heat transfer rates vs inclination at various Rayleigh numbers. -o- This work; □ Samuels and Churchill [10], (Ra = 3000, 4000); △ Wilkes and Churchill [8], (Ra = 2000).

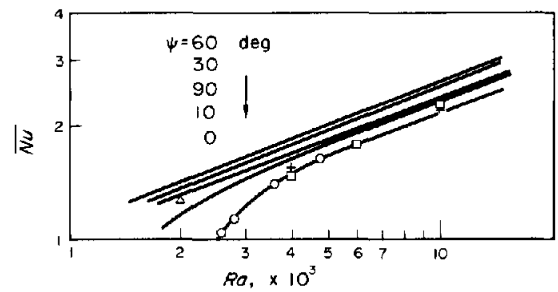

FIG. 3. Computed heat transfer rates vs Rayleigh number at various inclinations. — This work (cross-plotted from Fig. 2); △ Wilkes and Churchill [8], (90°); + MacGregor and Emery [9], (90°); □ Samuels and Churchill [10], (0°); o Ozoe and Churchill [7], (0°).

Ozoe and Churchill [7], Wilkes and Churchill [8], MacGregor and Emery [9], and Samuels and Churchill [10] computed the heat transfer rate at $\Psi = 90$ or 0 degrees of inclination for various Rayleigh numbers. (Those results which were reported only for finite grid-size were extrapolated to zero grid-size before comparison.) The values which were computed for the same Rayleigh numbers as in this investigation are included in Fig. 2. Those values which were computed for other Rayleigh numbers are compared with the cross-plotted curves for 0 and 90° inclination in Fig. 3. Agreement with all of the prior results is seen to be reasonably good.

4. EXPERIMENTAL APPROACH

Experiments were carried out to test the above predictions. The experimental apparatus is essentially the same as that used by Ozoe and Churchill [4, 5].

The flow pattern can be seen through transparent Plexiglas side walls (10 mm thick) which are covered by thermal insulation (Polystyrene plate) during the actual experiments. The upper side of the Plexiglas convection channel (inside dimensions $15 \times 15 \times 270$ mm) is a copper plate ($5 \times 120 \times 360$ mm) which is maintained at constant temperature by cooling-water circulating from a constant-temperature bath (circulation rate $1{\cdot}5 \times 10^{-4}$ m^3/s. The lower side of convection channel is also a copper plate ($10 \times 120 \times 360$ mm^3) which is heated by Nichrom wire (9 Ω). The heat input was measured by a watt meter. The usual input range was less than 10 W and the reading accuracy was $\pm 0{\cdot}1$ W. The input to the heating wire was maintained precisely by a constant-voltage regulator. The maximum DC-voltage was 10 and the regulating accuracy was ± 3 mV.

The temperature difference between the upper and lower plates was measured with a copper–constantan thermocouple. Thermocouple holes were drilled in the center of both copper plates. The thermocouple output was measured with an accuracy of $0{\cdot}1\,\mu$V ($0{\cdot}0024$°K) by a precision potentiometer.

The whole apparatus was placed in a constant-temperature room ($2\,\text{m} \times 2\,\text{m} \times 1{\cdot}5\,\text{m}$) maintained at $294{\cdot}15 \pm 0{\cdot}5$°K.

A graduator was attached on the T-shaped bar whose root was bolted on the upper copper plate. A plumb was hung from the center of the graduator for the measurement of the inclination of the apparatus from the horizontal plane. The whole apparatus was placed on a plate which was hinged to a large desk so that the inclination of the channel could be changed easily. Figure 4 is a schematic diagram of the experimental apparatus.

Glycerol was used as the experimental fluid. Aluminum powder was dispersed for observation of the flow pattern.

5. EXPERIMENTAL RESULTS

In the horizontal orientation of the square channel with heating from below and cooling from above, a series of two-dimensional roll-cells appeared with their axes perpendicular to the long dimension of the channel. When the channel was rotated 90 degrees, so that one vertical side was heated and the other cooled, a long two-dimensional roll-cell appeared. When the degree of inclination was decreased step-by-step from 90 degrees, the long two-dimensional cell kept its shape down to about 10 degrees. Between 10 degrees and zero degrees (horizontal), complex flow patterns appeared and suggested the possibility of multiple. stationary modes. When the channel was kept almost horizontal, with say less than 1 degree of inclination, a series of side-by-side two-dimensional roll-cells was eventually established. Thus a long two-dimensional roll-cell is the preferred mode in a square channel with an inclination greater than 10 degrees. Below 10 degrees side-by-side rolls or even more complex behavior is preferred. The critical degree of inclination seems to depend only slightly on the strength of convection, i.e. on the difference of temperature between the two plates.

Heat transfer rates were also measured. The temperature difference between the upper and lower copper plates usually attained a steady state in 18–29 ks after the change of electric voltage for the Nichrom wire. The angle of inclination was changed to 10, 20, ..., 90 degrees and the steady-state temperature-

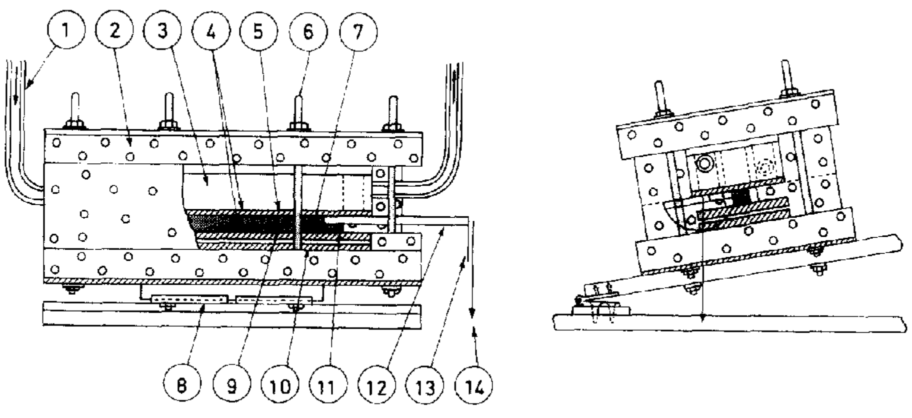

FIG. 4. Experimental apparatus: 1. Cooling water in; 2. Polystyrene thermal insulator; 3. Cooling water jacket; 4. Thermocouple holes; 5. Copper plate-cooling; 6. Brass bolt; 7. Experimental fluid; 8. Hinge; 9. Copper plate-heating; 10. Nichrom heater; 11. End of convection channel; 12. T-shape bar; 13. Graduator; 14. Plumb.

difference between the two plates was determined for each orientation. The heat loss was determined by heating from above with no motion. Rayleigh numbers were calculated using the physical properties at the mean temperature of the two plates. The Nusselt numbers measured for three Rayleigh numbers (and slightly differing mean-temperatures) are plotted versus the angle of inclination in Fig. 5. For all cases, the maximum Nusselt number appeared between 40 and 60 degrees of inclination. The experimental Nusselt numbers for $Ra = 3880$ and 4940 are slightly below the computed values for a single, long roll-cell but are in general agreement. The experimental values turn upward below 10 degrees because of the change in the mode of circulation. (The theoretical computations postulated no change in mode.) The experimental results for $\theta_{mean} = 309{\cdot}75$°K show more scatter, probably because the room temperature was not as well controlled for this case, resulting in greater heat loss to and heat input from the surroundings. For $\theta_{mean} = 301{\cdot}15$°K, △ indicates a change of inclination from 0 to 90 degrees and ▲ a change of inclination from 90 to 0 degrees. There seems to be no appreciable hysteresis.

Both the analytical and the experimental results suggest that the heat transfer rate goes through a maximum at about 50 degrees of inclination independent of the Rayleigh number.

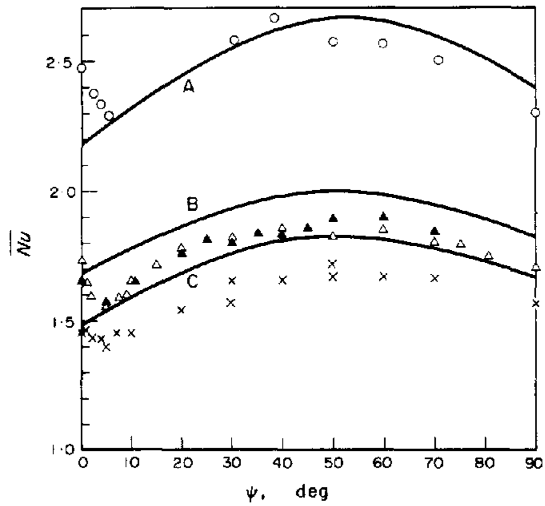

FIG. 5. Comparison of measured and computed heat transfer rates.

Code	Ra	Pr	°K	Experimental	Numerical
A	11 000	2690	309·75	·	—
B	4950	5220	301·15	△, ▲	—
C	3800	5580	300·45	X	—

6. CONCLUSIONS

The preferred mode of convection in a long, inclined, square channel was found to be a two-dimensional roll-cell with its axis in the long dimension of the channel for inclinations of the heated and cooled surfaces more than 10 degrees from the horizontal. As the degree of inclination decreases below 10 degrees multiple, stable states are apparently possible, but as the inclination approaches zero a series of two-dimensional roll-cells with their axes horizontal and perpendicular to the axis of the channel is eventually attained as the stable mode. The critical inclination for transition between the modes depends slightly on the temperature-difference between two plates.

The heat transfer rate was observed both experimentally and theoretically to attain a maximum with the heating and cooling surfaces inclined about 50 degrees from the horizontal. Thus the rate of convection can be increased relative to either heating from below or heating from the side by rotating the channel to an intermediate inclination. This same tendency was predicted theoretically by Holst and Aziz [2] for a porous media but not tested experimentally. The heat transfer rate was observed experimentally to go through a minimum when the heating and cooling surfaces were inclined about 10° from the horizontal. Thus the rate of convection can be decreased slightly by rotating the channel about its axis.

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CONVECTION NATURELLE DANS UN CANAL CARRE INCLINE

Résumé—Un bon accord a été établi entre les valeurs expérimentales et les valeurs calculées du nombre de Nusselt relatif au transfert thermique par convection naturelle dans un canal incliné, à section carrée, avec des faces maintenues à des températures uniformes différentes.

La théorie et l'expérience montrent ensemble que le flux maximal de chaleur se produit pour une inclinaison de 50 degrés environ. Pour une inclinaison de 10° environ, on obtient expérimentalement le flux thermique minimal et un changement d'orientation des cellules tourbillonnaires bidimensionnelles.

FREIE KONVEKTION IN EINEM GENEIGTEN QUADRATISCHEN KANAL

Zusammenfassung—Experimentelle und numerische Untersuchungen wurden zur Ermittlung der Nusselt-Zahl durchgeführt für den Wärmetransport bei freier Konvektion in einem geneigten quadratischen Kanal. Dabei waren die einander gegenüberliegenden geneigten Seiten von einheitlicher aber unterschiedlicher Temperatur. Es ergab sich gute Übereinstimmung. Das Maximum für den Wärmestrom wurde sowohl theoretisch als auch experimentell für einen Neigungswinkel von etwa 50° gefunden. Der minimale Wärmestrom und eine Änderung in der Orientierung der zwei-dimensionalen Roll-Zellen tritt nach experimentellen Untersuchungen bei einem Neigungswinkel von etwa 10° auf.

ЕСТЕСТВЕННАЯ КОНВЕКЦИЯ В НАКЛОННОМ КАНАЛЕ КВАДРАТНОГО СЕЧЕНИЯ

Аннотация — В результате численного решения и экспериментального исследования определены значения числа Нуссельта для теплообмена при естественной конвекции в наклонном канале квадратного сечения при однородной температуре на одной наклоненной стороне и при более низкой однородной температуре на противоположной стороне, и найдено их соответствие. Как теоретически, так и экспериментально найдено, что максимальный коэффициент теплообмена соответствует $\sim 50^\circ$ наклона. Экспериментально найдено, что минимальный коэффициент теплообмена и изменение ориентации двумерных валообразных ячеек имеют место при наклоне $\sim 10^\circ$.